
ELASTIC WAVES FROM A DISTRIBUTED SURFACE SOURCE IN A UNIDIRECTIONAL COMPOSITE LAMINATE

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ABSTRACT

The response of a unidirectional composite plate of infinite lateral dimensions to localized dynamic surface sources is investigated through theoretical modeling and laboratory tests. In the theoretical simulations, the material of the plate is assumed to be dissipative and transversely isotropic with its symmetry axis parallel to the fiber. The source is assumed to have an arbitrary spatial and time dependence. The associated elastodynamic boundary value problem is solved by means of an integral transform technique followed by numerical evaluation of the inversion integrals. The laboratory tests are carried out on unidirectional graphite epoxy plates of thicknesses ranging from 1-25 mm and large lateral dimensions ($> 30 \text{ cm}^2$) excited by means of broadband transducers attached to its surface. The calculated surface response of the plate at different distances and directions from the source is shown to agree very well with the recorded response in the ultrasonic range.

INTRODUCTION

It is well known that laminated fiber reinforced composites often suffer significant internal damage when they are subjected to localized dynamic surface loads. The damage may involve fiber breakage and debonding as well as delamination between the individual laminas. Such damage has been observed to occur even at relatively low impact speeds resulting in a severe loss in the load carrying capacity of the laminas. Although the damage is clearly caused by the stresses which develop within the material, the precise nature of these stresses and their relationship to the degree and mode of the damage are not clearly understood at present. This is particularly true in the dynamic case where the stresses are caused by waves whose propagation characteristics are strongly influenced by the inherent anisotropy and heterogeneity of the composite material.

Dynamic response of plates has been studied theoretically by many authors through past decades. The linear elastic solutions of the isotropic or anisotropic plates have been investigated by, e.g., Weaver and Pao (1982), Vasudevan and Mal (1985); Xu and Mal (1987), Liu et al. (1991a, b), Mal and Lih (1992, 1995). For low frequency response quasi-static and thin plate theories have been used (see e.g., Chow (1971), Moon (1973); Sun and Tan (1984); Lal (1984)). While numerous analytical investigation of wave propagation in plate have been executed, concurrent experimental studies of such wave processes in the laboratory have been far less prevalent. Most effort in this direction were devoted to a comparison of predicted phase and group velocities of surface waves.

for waveform analysis, Medick (1960) used a 220 Swift rifle bullet impacting perpendicular to an aluminum plate to generate flexure waves. Recently, German et al. (1989) used a lead break on the surface of a plate to generate waves; however, there are no available results for comparison between the measured and calculated time history for wave propagation in a composite under dynamic surface loads.

In this paper a classical integral transform technique coupled with the matrix method developed by Mal and Lih (1992) is used for the theoretical simulation of the surface source problem. The integrals involved in the spatial inverse transform are evaluated by means of a previously developed adaptive integration scheme. The measurements are earned out by means of an ultrasonic system called the Fracture Wave Detector by Digital Wave Corp. Numerical and experimental results for the response of a unidirectional composite laminate due to a quasi-sine pulsed load are compared.

THEORY

Material Modeling

Composite materials are known to be strongly anisotropic as well as dissipative. In fiber-reinforced composites dissipation is caused by the anisotropy of fiber orientations, and the dissipation of the waves is caused by the viscoelastic nature of the resin and by scattering from the fibers and other inhomogeneities. Both effects can be modeled in the frequency domain by assuming that the stiffness constants, C_{ij} , are complex and frequency-dependent. A possible form of C_{ij} that can model the essential features of the dissipation caused by these factors, has been given in Mal, Bar-Cohen, and Lih (1992) and will be used here. A brief description of the model is presented for completeness; the details can be found in the cited paper.

We recall that the linear constitutive equation for a transversely isotropic elastic solid with its symmetry axis along the x_1 -axis (Figure 1) can be expressed in the form

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{22} & 0 & 0 & 0 \\ c_{12} & c_{23} & c_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{55} \end{bmatrix} \begin{Bmatrix} u_{1,1} \\ u_{2,2} \\ u_{3,3} \\ 2u_{2,3} + u_{3,2} \\ u_{1,3} + u_{3,1} \\ u_{1,2} + u_{2,1} \end{Bmatrix} \quad (1)$$

where $c_{44} = (c_{22} - c_{23})/2$, σ_{ij} is the Cauchy's stress tensor, u_i is the displacement vector and $c_{11}, c_{12}, c_{22}, c_{23}, c_{55}$ are the five independent real stiffness constants of the material. We introduce five additional constants a_1, a_2, a_3, a_4 and a_5 related to C_{ij} and the density of the material, ρ , through

$$\begin{aligned} a_1 &= c_{22}/\rho, a_2 = c_{11}/\rho, a_3 = (c_{12}^* + c_{55})/\rho \\ a_4 &= (c_{22} - c_{23})/2\rho = c_{44}/\rho, a_5 = c_{55}/\rho \end{aligned} \quad (2)$$

It is well known that the quantities $\sqrt{a_1}, \sqrt{a_2}, \sqrt{a_3}, \sqrt{a_4}$ and $\sqrt{a_5}$ represent the velocities of five independent bulk waves that can be transmitted along certain specific directions in the transversely isotropic solid.

Let $C_{11}, C_{22}, C_{44}, C_{55}$ denote the complex and frequency-dependent stiffness constants of the fiber-reinforced composite and let the complex constants A_1, A_2, A_3, A_4 be A_5 are defined through,

$$\begin{aligned} A_1 &= C_{22}/\rho, A_2 = C_{11}/\rho, A_3 = (c_{12}^* + C_{55})/\rho, \\ A_4 &= C_{44}/\rho, A_5 = C_{55}/\rho \end{aligned} \quad (3)$$

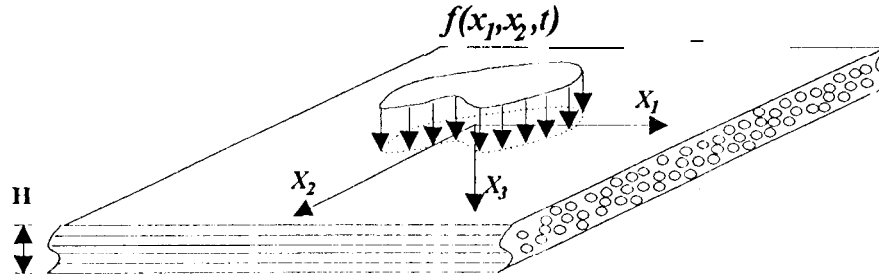


FIGURE 1. THE SURFACE LOAD PROBLEM FOR A UNIDIRECTIONAL COMPOSITE

We assume that the two sets of constants A_i and a_i are related by the equations

$$\begin{aligned} A_1 &= a_1 / (1 + ip\sqrt{a_3/a_1}), & A_2 &= a_2 / (1 + ip\sqrt{a_3/a_2}), \\ A_3 &= a_3 / (1 + ip\sqrt{a_3/a_3}), & A_4 &= a_4 / (1 + ip\sqrt{a_3/a_4}), \\ A_5 &= a_5 / (1 + ip) \end{aligned} \quad (4)$$

where p is the damping factor (Mal et al, 1992). For many isotropic solids, p is independent of frequency in a broad frequency range. This is generally true if the wavelengths are long compared to the microstructural dimensions (e.g., grain size) of the solid. At higher frequencies the damping factor increases with frequency due to wave scattering. We assume that p is constant below a certain frequency ω_0 , beyond which it becomes frequency dependent and that it can be expressed in the form

$$p = p_0 [1 + a_0 (\frac{f}{f_0} - 1)^2 H(f - f_0)] \quad (5)$$

where $f = \omega/2\pi$ is the frequency in cycles, $f_0 = \omega_0/2\pi$, $H(f)$ is the Heaviside step function, and a_0, p_0, a_0, f are constants which determine the degree of decay in the amplitude of the waves with propagation distance.

The first term in the right hand side of (5) represents dissipation due to internal friction and other thermodynamic effects, while the second term represents the attenuation caused by wave scattering by the fibers and other inhomogeneities in the material; moreover, wave attenuation due to scattering becomes more and more prominent as $f - f_0$ increases.

The material model has been used to calculate the reflected field from graphite/epoxy plates of different thicknesses immersed in water and the results have been compared with measured data in a pitch-catch type arrangement in an ultrasonic test bed (Mal et al. (1992)). The predicted results have been found to be in remarkable agreement with the measured data. We now use this model to calculate the elastodynamic field produced by a concentrated surface load on a graphite/epoxy laminate.

The Surface Load Problem

The formal solution for a concentrated load problem can be found in (Mal and Lih (1992)). The general procedures are described briefly as follows. Consider a unidirectional composite laminate with thickness H and with fiber directions parallel to x_1 axis. The applied load $f(x, x, t)$ is assumed to be acting at the top surface (Figure 1). In absence of body forces, the governing equations of the problem become

$$\sigma_{ij} = \rho u_{i,j} \quad (6)$$

The boundary conditions can be expressed as

$$\begin{aligned} \sigma_{i3}(x_1, x_2, 0, t) &= f_i(x_1, x_2, t) \\ \sigma_{i3}(x_1, x_2, H, t) &= 0 \end{aligned} \quad (7)$$

where $i = 1, 2, 3$. In order to solve the problem, we introduce Fourier time transforms of all time-dependent variables f through

$$f(x_1, x_2, t) = \operatorname{Re} \frac{1}{\pi} \int_0^\infty \hat{f}(x_1, x_2, \omega) e^{-i\omega t} d\omega \quad (8)$$

$$\hat{f}(x_1, x_2, \omega) = \int_0^\infty f(x_1, x_2, t) e^{i\omega t} dt \quad (9)$$

and denote the Fourier time transform of the displacement and stress components $u_i(x, t), \sigma_{ij}(x, t)$ by $\hat{u}_i(x, \omega), \hat{\sigma}_{ij}(x, \omega)$. Then $\hat{u}_i(x, \omega), \hat{\sigma}_{ij}(x, \omega)$ are solutions of the system

$$\hat{\sigma}_{ij} + \rho \omega^2 \hat{u}_i = 0 \quad (10)$$

$$\hat{\sigma}_{i3}(x_1, x_2, 0, \omega) = \hat{f}_i(x_1, x_2, \omega) \quad (11)$$

$$\hat{\sigma}_{i3}(x_1, x_2, H, \omega) = 0, \quad (i = 1, 2, 3) \quad (12)$$

where $f_i(x_1, x_2, \omega)$ is the Fourier time transform of $f_i(x_1, x_2, t)$. The Cauchy's equations of motion (Equation 10) must be supplemented by the constitutive Equation (1) and the solution must satisfy the outgoing wave (or radiation) condition at large lateral distances from the load. In order to obtain a formal solution of the boundary-value problem in the frequency domain, we introduce the double spatial Fourier transforms of $\hat{u}_i(x, \omega), \hat{\sigma}_{ij}(x, \omega)$, and $f_i(x_1, x_2, \omega)$ through

$$\hat{u}_i(x_1, x_2, x_3, \omega) = \frac{1}{4\pi^2} \int_{-\infty}^\infty \int_{-\infty}^\infty U_i(\xi_1, \xi_2, x_3, \omega) e^{i(\xi_1 x_1 + \xi_2 x_2)} d\xi_1 d\xi_2 \quad (13)$$

$$\hat{\sigma}_{ij}(x_1, x_2, x_3, \omega) = \frac{1}{4\pi^2} \int_{-\infty}^\infty \int_{-\infty}^\infty \Sigma_{ij}(\xi_1, \xi_2, x_3, \omega) e^{i(\xi_1 x_1 + \xi_2 x_2)} d\xi_1 d\xi_2 \quad (14)$$

$$\hat{f}_i(x_1, x_2, \omega) = \frac{1}{4\pi^2} \int_{-\infty}^\infty \int_{-\infty}^\infty F_i(\xi_1, \xi_2, \omega) e^{i(\xi_1 x_1 + \xi_2 x_2)} d\xi_1 d\xi_2 \quad (15)$$

To solve the problem, a six-dimensional "stress-displacement vector" $\{S\}$ in the transformed domain is

introduced,

$$\{S(x_3)\} = \{U_i(x_3) \Sigma_{i3}(x_3)\} \quad (16)$$

Then $\{S\}$ can be expressed in a partitioned matrix product form as

$$\{S(x_3)\} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} E^+(x_3) & 0 \\ 0 & E^-(x_3) \end{bmatrix} \begin{Bmatrix} C^+ \\ C^- \end{Bmatrix} \quad (17)$$

where

$$E^\pm(x_3) = \text{diag}[e^{\pm i\zeta_1 x_3} \quad e^{\pm i\zeta_2 x_3} \quad e^{\pm i\zeta_3 x_3}] \quad (18)$$

In the above, c, c' are complex constants related to downgoing and upgoing waves within the laminate, $[Q_{ij}]$ are 3×3 matrices and ζ_i are the "vertical" wave numbers of the three possible waves in the composite with "horizontal" wave numbers ξ_1, ξ_2 . The expressions for $[Q_{ij}]$ and ζ_i are given in the Mal and Li (1992). Then the six constants can be determined from the boundary conditions. The solutions for the top and bottom displacement can then be expressed as

$$\{U(0)\} = [Q_{11} + Q_{12} E Q_{22}^{-1} Q_{21} E] [Q_{21} + Q_{22} E Q_{22}^{-1} Q_{21} E]^{-1} \{\Sigma(0)\} \quad (19)$$

$$\{U(H)\} = [Q_{11} - Q_{12} E Q_{22}^{-1} Q_{21} E] [Q_{21} - Q_{22} E Q_{22}^{-1} Q_{21} E]^{-1} \{\Sigma(0)\} \quad (20)$$

where $\{\Sigma(0)\}$ is the stress vector on the top surface of the plate. We assume that load is normal to the surface of the laminate, and that the applied load can be separated into a time dependent function and spatially distributed function $p(x_1, x_2)$. Denote the Fourier time transform of $f(t)$ by $f(\omega)$, and the spatial double Fourier transform of $p(x_1, x_2)$ by $P(\xi_1, \xi_2)$. Then $\{W\}$ becomes

$$\{\Sigma(0)\} = -\hat{f}(\omega) (0, 0, P(\xi_1, \xi_2)) \quad (21)$$

where

$$P(\xi_1, \xi_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x_1, x_2) e^{-i(\xi_1 x_1 + \xi_2 x_2)} dx_1 dx_2 \quad (22)$$

In this paper two specific type of are concerned: one is a unit concentrated load (Figure 2(a)), where $p(x_1, x_2) = \delta(x_1)\delta(x_2)$ so that $P(\xi_1, \xi_2) = 1$. The other is a unit load uniformly distributed in a circular region with radius a (Figure 2(b)) with,

$$\begin{aligned} P(\xi_1, \xi_2) &= \int_0^a \int_0^{2\pi} e^{ir(\xi_1 \cos \theta + \xi_2 \sin \theta)} dr d\theta \\ &= \frac{2\pi a}{\xi} J_1(\xi a), \quad (\xi \neq 0) \\ &= \pi a^2, \quad (\xi = 0) \end{aligned} \quad (23)$$

where $\xi = \sqrt{\xi_1^2 + \xi_2^2}$ and $J_1(x)$ is the Bessel function of the first kind of order 1. A recently developed adaptive

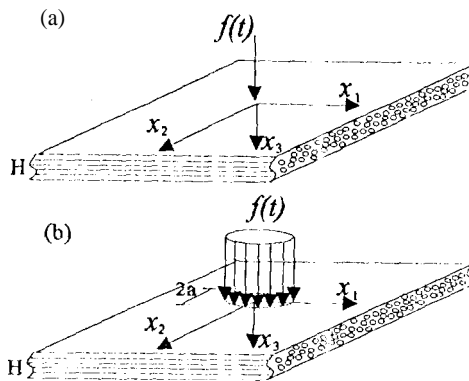


FIGURE 2. (a) A POINT LOAD (b) A DISTRIBUTED LOAD IN A CIRCULAR AREA

numerical integration scheme (Mal and Lih, 1992) has been used to evaluate the wavenumber integral and the resulting spectra are inverted by fast Fourier transform (FFT).

THE EXPERIMENT

The experimental setup is shown in Figure 3. The source is a single pulse generated by a Stanford DS345 function generator and amplified by a Ritec A300 RFG gated pulse amplifier. Identical broadband transducers (Digital Waves, Model 131000, 5 MHz) were used as transmitter and receiver. A Fracture Wave Detector (Digital Waves, F4000) with four signal conditioning modules was used for data acquisition. The modules are integrated with triggers, trigger threshold, echo delay controller, filters, and AD converters which can acquire data at rates

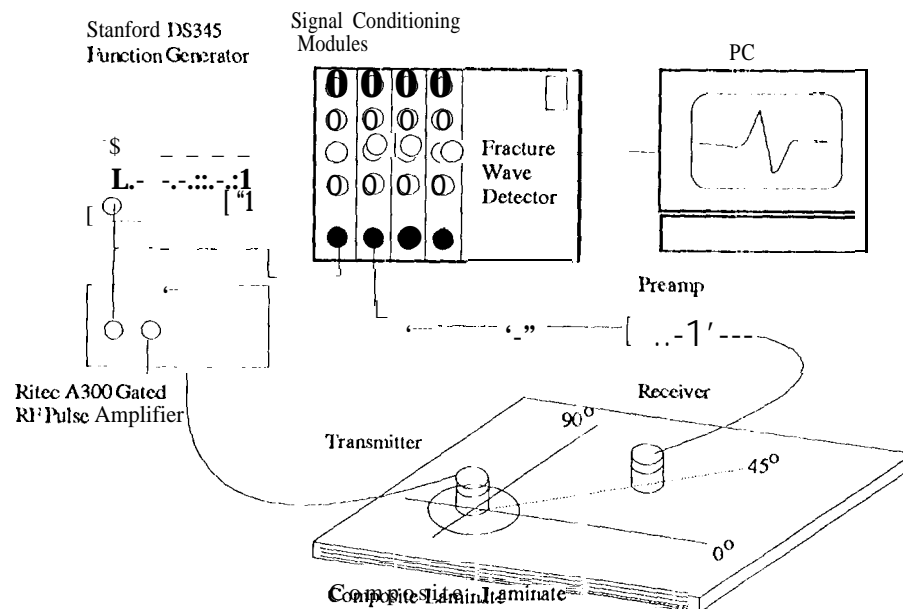


FIGURE 3. THE EXPERIMENTAL SETUP

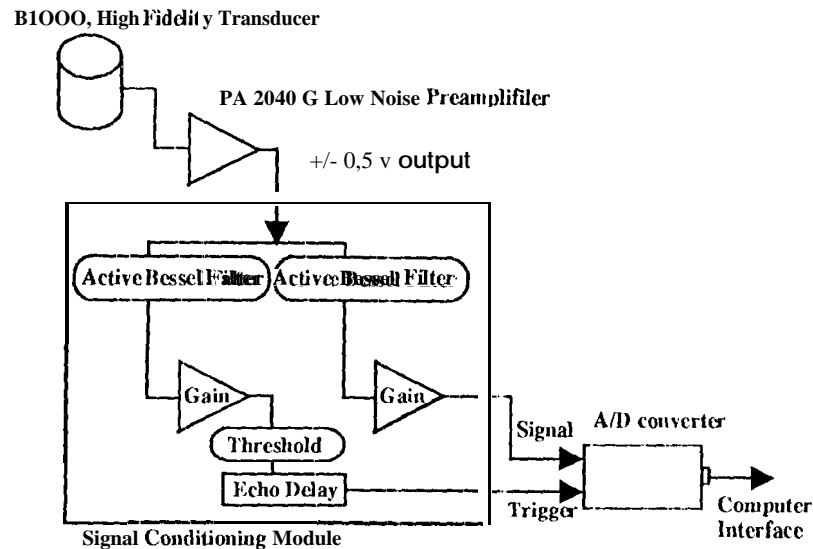


FIGURE 4. F4000 FRACTURE WAVE DETECTOR BLOCK DIAGRAM

from 3.715 to 25 MHz. The block diagram of the Fracture Wave Detector is shown in Figure 4. As depicted in Figure 3, the transducers were connected to wideband preamplifiers then to signal conditioning modules to record and digitize the signals at a rate of 12.5 MHz. The data for each signal were transferred to a personal computer for analysis. The measurements were made for filed points at 0°, 45° and 90° to the fibers,

A $[0]_{16}$ 12 x 12 cm² unidirectional graphite/epoxy plate was used in the experiment. The material used was AS4/3502. The plate was made in UCI A's Composite Manufacturing Lab. The laminate thickness was 3.175 mm, with stiffness constants $c_{11} = 155.01$, $c_{12} = 6.44$, $c_{21} = 15.6$, $c_{33} = 7.89$, $c_{44} = 5.00$ (GPa), and density $\rho = 1.56$ g/cm³. The damping coefficients were assigned the values with $p_0 = 0.005$, and $a_0 = 0$.

RESULTS

The calculations and the measurements were carried out for a variety of specimens and sources to station distances. Here we present results for a case in which the distance of the field point is much larger than the source dimensions.

The result for wave propagation in aluminum plate is first presented for comparison. The material constants for aluminum are $E = 72.48$ GPa, $\mu = 26.91$ GPa, and the damping constants used are $p_0 = 0.005$, $a_0 = 0$. The source function is plotted in Figure 5. It should be noted that the source function generated from the function generator is originally a 200 KHz single sine pulse; however, through gating and amplifying together with the transformation through transducers, the original source function is altered.

Figure 6 shows the comparison between measured and calculated time histories of normal displacement u_3 on a 3.175 mm thick aluminum plate at 50 mm from the source. It can be seen that there is excellent agreement between the calculated and the measured waveforms. Figure 7 shows the comparison between measured and calculated time histories of the normal displacement u_3 at 30 mm from source on a 3.175 mm thick graphite/epoxy laminate with material constants mentioned above for wave propagation along the fiber. Although the agreement is not as excellent as that with aluminum, it is very good. Figure 8 is the same case as Figure 7 for propagation 45° to the fiber, although there is some difference for the first arrivals, the main pulses are in good agreement. Figure 9 is the same as Figure 7 for wave propagation 90° to the fiber. The result again shows good agreement between the measured and calculated results. Note that the differences at the end of the time history is caused by reflections from the boundaries of the plate.

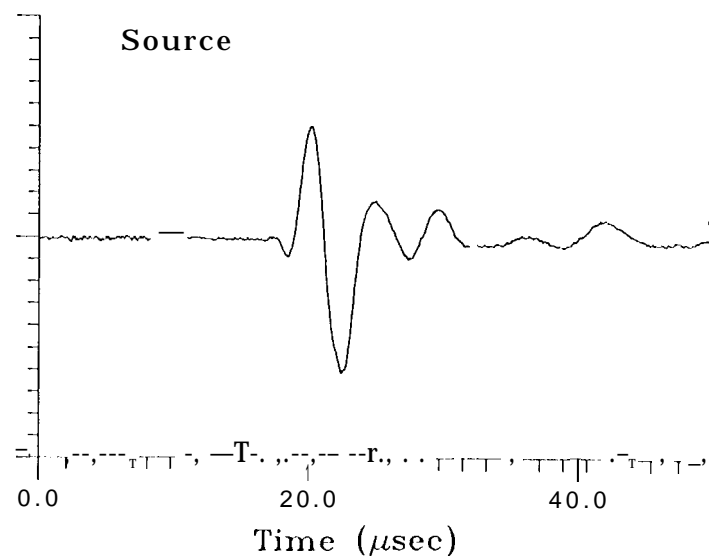


FIGURE 5. TIME HISTORY OF SOURCE

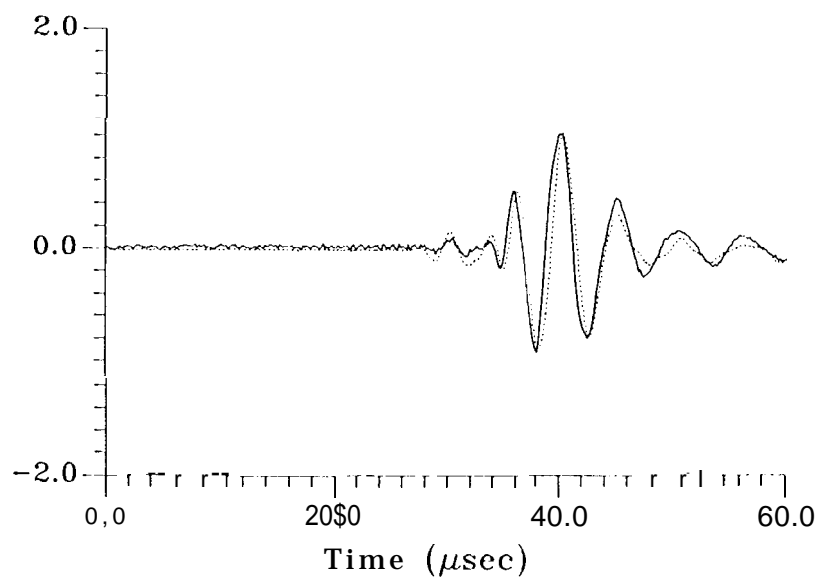


FIGURE 6. COMPARISON BETWEEN MEASURED (DASHED LINE) AND CALCULATED (SOLID LINE) TIME HISTORIES OF NORMAL DISPLACEMENT U , ON A 3.175 MM THICK ALUMINUM PLATE AT 50 MM FROM SOURCE

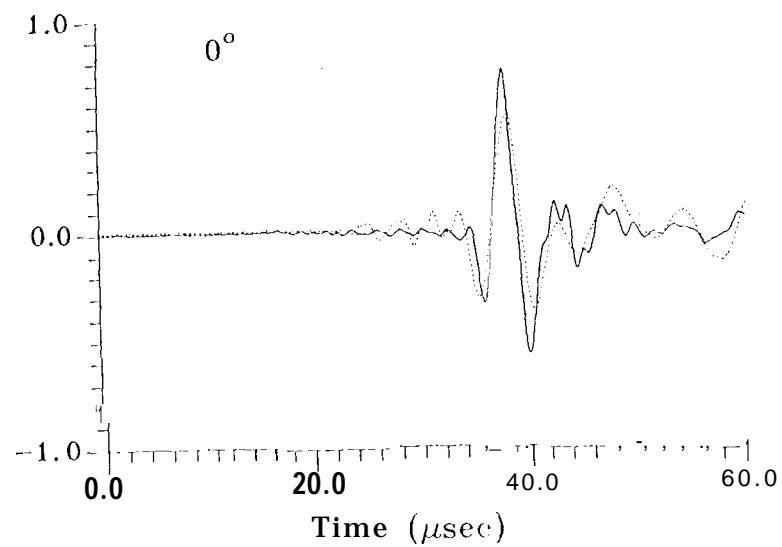


FIGURE 7. COMPARISON BETWEEN MEASURED (DASHED LINE) AND CALCULATED (SOLID LINE) TIME HISTORIES OF THE NORMAL DISPLACEMENT U_3 , ON A 3.175 MM THICK GRAPHITE/EPOXY LAMINATE PLATE AT 30 MM FROM SOURCE FOR WAVE PROPAGATION ALONG 0° TO THE FIBER

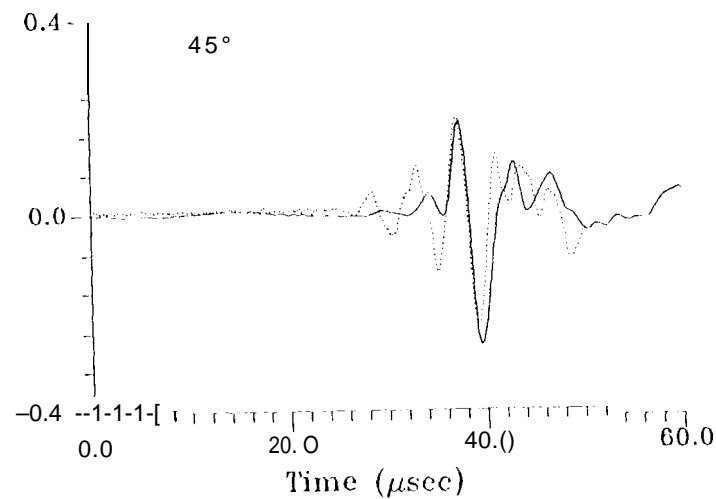


FIGURE 8. SAME AS IN FIGURE 7 FOR WAVE PROPAGATION 45° TO THE FIBER

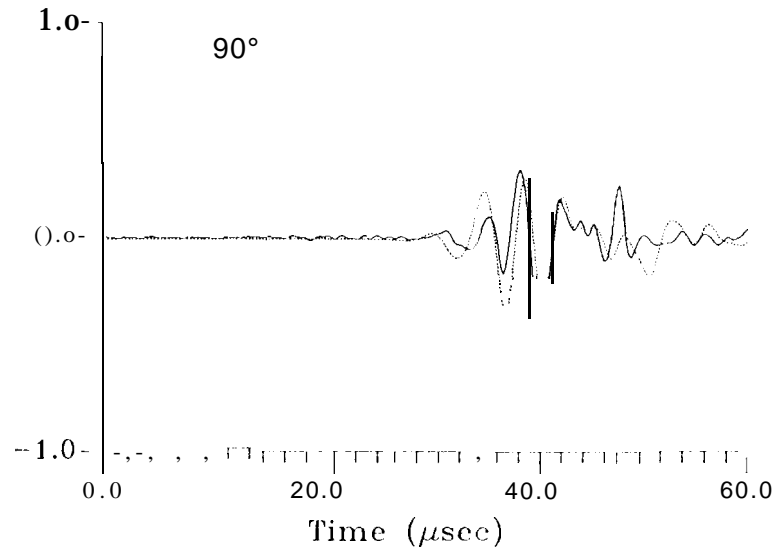


FIGURE 9. SAME AS IN FIGURE 7 FOR WAVE PROPAGATION 90° TO THE FIBER.

CONCLUDING REMARKS

An investigation comprised of theoretical analysis and experiments was performed to study the response of composite laminates to dynamic surface loads. It shows good agreement of the main pulses between the measured data and the calculated results; however the agreement between calculated and measured time history is excellent for the isotropic plate, but it is reduced somewhat for the composite laminate. Improvements in the model and experiment are needed to obtain better results. The quantitative features of a wave propagating in a composite laminate are better understood. The understanding developed in this work will be useful for the prediction of the damage produced in composite laminates subjected to low speed impacts and for the ultrasonic NDE of structural composite components.

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